FOM 12

1.3 – Compound Interest: FUTURE VALUE

Compound Interest – The interest that is earned on **both** the principal **and** the accumulated interest.

Future Value (FV or A) – How much an investment is worth after a certain amount of time.

Compounded Annually – When compound interest is determined or paid **yearly**.

Example 1:

Yvonne earned $4300 in overtime on a carpentry job. She invested the money in a 10-year Canada Savings Bond that will earn 3.8% **compounded annually**. She decided to invest in a CSB, instead of keeping the money in a savings account, because the CSB will earn more interest. What is the future value of Yvonne's investment after 10 years?

$P = \$4300 \quad t = 10 \quad r = 0.038$

Work out the amount at the end of each year. Is there a pattern? Can you create a formula that will work for any compound interest formula?

First Year $FV = P + Prt = P(1+rt)$

$= 4300 + 4300(0.038)(1) = \$4463.40$ ← New Principal

2nd Year $FV = P + Prt$

$= 4463.40 + 4463.40(0.038)(1) = \$4633.01$

3rd Year $FV = P + Prt$

$= 4633.01 + 4633.01(0.038)(1) = \$4809.06$

⋮

Too Long!

First Year: $FV = P + Prt = P(\underbrace{1+rt}_{1.038})$

$FV = 4300(1 + (0.038)(1))$

$= 4300 \times 1.038 = \$4463.4$

2nd Year: $FV = P \times 1.038 = \$4633.01$

3rd Year: $= P \times 1.038 = \$4809.06$

4th Year: $= P \times 1.038 = \$4809.06$

Each year just multiplying by 1.038 (1+rt)

$FV = P \times 1.038 \times 1.038 \times 1.038 \times \ldots = P \times 1.038^{10}$

$= 4300 \times 1.038^{10}$ $\boxed{= \$6243.70}$

Formula is a bit more complicated than Example 1 because Interest isn't always compounded annually.

Compound Interest Formula: $FV = P(1+i)^n$

FV = Amount the investment is worth in the end (the future value)
(also shown as A)

P = The Principal (Starting Amount)

i = interest per compounding period $= \frac{rate}{\#\ times\ compounded\ per\ year}$

n = Number of compoundings during the investment

$$n = (\#\ times\ compounded\ per\ year) \times (\#\ years)$$

Example 2: Matt has invested \$23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 5 to 10 years.

a) What is the future value of the investment after 5 years? What is the future value after 10 years?

5-Years

$P = 23000$

$i = \frac{0.136}{2} = 0.068$

$n = 2 \times 5 = 10$ $\quad A = P(1+i)^n$

$FV = 23000(1+0.068)^{10}$

$FV = 23000(1.068)^{10}$

$FV = 23000(1.93068991)$

$\boxed{FV = \$44405.87}$

10-Years

$P = 23000$

$i = \frac{0.136}{2} = 0.068$

$n = 2 \times 10 = 20$ $\quad A = P(1+i)^n$

$FV = 23000(1+0068)^{20}$

$FV = 23000(1.068)^{20}$

$FV = 23000(3.72756356$

$\boxed{FV = \$85733.96}$

b) If the investment had earned simple interest, would the relationship between the principal and the future values have been the same? Explain.

No. Simple Interest is only calculated on the Principal.

Example: In 10 years the Interest would be

$I = 23000(0.136)(10) = \$31280$

$FV = 23000 + 31280$

$\boxed{= \$54280}$

Example 3: Both Joli, age 50, and her daughter Lena, age 18, plan to invest $1500 in an account with an annual interest rate of 9%, **compounded monthly**.

a) If both women hold their investments until age 65, what will be the difference in the future values of their investments?

Joli

How long is investment?

$65-50=15$ years

$P=1500$

$i=\frac{0.09}{12}=0.0075$

$n=12\times15=180$

$A=1500(1+0.0075)^{180}$

$A=1500(3.8380\ldots)$

$A=\$5757.06$

Lena

How long is investment?

$65-18=47$

$P=1500$

$i=\frac{0.09}{12}=0.0075$

$n=12\times47=564$

$A=1500(1+0.0075)^{564}$

$A=1500(67.6411\ldots)$

$A=\$101461.71$

Find the Difference:

$101461.71-5757.06$

$\boxed{=\$95704.65}$

b) Lena's older step-brother Cody, age 34, also plans to invest $1500 at 9% compounded monthly. Determine the future value of his investment at age 65.

Length of Investment: $65-34=31$

$P=1500$

$i=\frac{0.09}{12}=0.0075$

$n=31\times12=372$

$FV=P(1+i)^n$

$FV=1500(1.0075)^{372}$

$\boxed{FV=\$24168.61}$

Rule of 72 – A simple formula for estimating the doubling time of an investment.
(most accurate when compounded annually)

$$\boxed{\textbf{Number of Years to Double} = 72 \div \textbf{Interest Rate as \%}}$$

Example 5: Both Berta and Kris invested $5000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.

Berta

$\#\text{ Years} = 72 \div 8$

$= 9$ years

Kris

$\#\text{ Years} = 72 \div 9$

$= 8$ years

8% doubles every 9 years (see above)

b) Estimate the future value of an investment of $5000 that earns 8%, compounded annually, for 9, 18, and 27 years. How close are your estimates to the actual future values?

$A = P(1+i)^n$

$P = 5000$
$i = \frac{0.08}{1} = 0.08$
$n = 1 \times 9$

$A = 5000(1+0.08)^9$
$= 5000(1.999004627) = \$9995.02$

9 years

$P = 5000$
$i = 0.08$
$n = 1 \times 18$

$A = 5000(1.08)^{18}$
$= 5000(3.996019499) = \$19980.10$

18 years

$P = 5000$
$i = 0.08$
$n = 1 \times 27$

$A = 5000(1.08)^{27}$
$= 5000(7.988061469) = \$39940.31$

27 years

Very close to estimates of $10000, $20000 & $40000

Example 5: <u>Estimate</u> how long it would take for $1000 to grow to $16 000 at an interest rate of 6% compounded semi-annually.

$P = 1000$

$FV = 16\,000$

$i = \frac{0.06}{2} = 0.03$

$n = ?$

$FV = P(1+i)^n$

$$\frac{16000}{1000} = \frac{1000\boxed{(1.03)^n}}{1000}$$

these stay together!

$16 = 1.03^n$

use trial & error to find value of n that gives answer of 16 (or just over 16)

$1.03^{15} = 1.55797$ (way too small)

$1.03^{50} = 4.384$ (still too small)

$1.03^{100} = 19.219$ (too big)

$1.03^{93} = 15.627$

$\boxed{1.03^{94} = 16.095}$

this means that $n = 94$

n is also the # years x # compoundings per year.

so $\# \text{ years} = \frac{n}{\#\text{compoundings per year}}$

$t = \frac{94}{2} = 47$

It will take about 47 years

Assignment: Pg. 30 #2, 5-12 (Extension # 14-15)