

1.4 – Compound Interest: PRESENT VALUE

Compound Interest – The interest that is earned on **both** the principal and the accumulated interest

Present Value – The amount that must be invested now to result in a specific future value. (Really it is just the **PRINCIPAL**)

Compound Interest Formula: $FV = P(1 + i)^n$

FV = Amount the investment is worth in the end. The future value (also known as)

P = The Principal (Starting Amount) → Present Value.

i = interest per compounding period = $\frac{\text{rate}}{\# \text{ times compounded per year}}$

n = Number of compoundings during the investment

$n = (\# \text{ times compounded per year}) \times (\# \text{ years})$

Example 1: What is the present value of \$5860.13 if it was invested at 3.2%, compounded semi-annually for 5 years?

$$FV = \$5860.13$$

$$P = ?$$

$$i = \frac{0.032}{2} = 0.016$$

$$n = 2 \times 5 = 10$$

$$FV = P(1+i)^n$$

$$5860.13 = P(1+0.016)^{10}$$

$$5860.13 = P(1.016)^{10}$$

$$5860.13 = P(1.17202555)$$

$$1.17202555$$

$$1.17202555$$

$$\$5000.00 = P$$

Use "ANS" button on calculator

The present value (Principal)
was \$5000

Example 2: Ginny is 18 years old. She has inherited some money from a relative. Ginny wants to invest some of the money so that she can buy a home in Mile River, Alberta, when she turns 30. She estimates that she will need about \$170 000 to buy a home. How much does she have to invest now, at 6.5% compounded annually? (What is the present value?)

Time: $30 - 18 = 12$ years

$$FV = 170000$$

$$i = \frac{0.065}{1} = 0.065$$

$$n = 1 \times 12 = 12$$

$$FV = P(1+i)^n$$

$$170000 = P(1+0.065)^{12}$$

$$\frac{170000}{1.065^{12}} = \frac{P(1.065)^{12}}{1.065^{12}}$$

$$\$79846.09 = P$$

She should invest \$79 846.09

Example 3:

Laura has invested \$15 500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least \$50 000 by the time her newborn enters university, in 18 years.

a) What interest rate, compounded annually, will result in a future value of \$50 000? Round your interest rate to two decimal places.

$$P = 15500$$

$$FV = 50000$$

$$i = ?$$

$$n = 1 \times 18 = 18$$

$$FV = P(1+i)^n$$

$$\frac{50000}{15500} = \frac{15500(1+i)^{18}}{15500} \rightarrow \text{keep together}$$

$$3.225806452 = (1+i)^{18}$$

$$\sqrt[18]{\text{ANS}} = \sqrt[18]{(1+i)^{18}}$$

$$1.067229 = 1+i$$

$$0.067229 = i$$

(compounded annually so $i=r$)

Interest Rate is 6.72%

b) Suppose that Laura wants her \$15 500 to grow to at least \$60 000 at the interest rate from part a. How long will this take?

$$FV = 60000$$

$$P = 15500$$

$$i = 0.0672$$

$$n = ?$$

$$\frac{60000}{15500} = \frac{15500(1.0672)^n}{15500}$$

$$3.870967742 = 1.0672^n$$

Trial & Error to Find n.

$$1.0672^5 = 1.384 \text{ (small)}$$

$$1.0672^{10} = 1.916 \text{ (small)}$$

$$1.0672^{20} = 3.672 \text{ (small)}$$

$$1.0672^{21} = 3.9188 \text{ (big)}$$

> 3.87 is between. so have to use $n=21$

Compounded annually so $n = \text{years}$

∴ 21 years