

1.5 – Investments Involving Regular Payments

Compound Interest Formula: $FV = P(1 + i)^n$

Example 1: An investment is made by depositing \$500 every 6 months into an RRSP (Registered Retirement Savings Plan) at 6% compounded semi-annually for 20 years. What is the Final Value?

$FV = P(1+i)^n$ $i = \frac{0.06}{2} = 0.03$

1st Deposit \downarrow 2×20
 2nd Deposit \downarrow halfway through first year
 3rd Deposit 4th

$500(1.03)^{40} + 500(1.03)^{39} + 500(1.03)^{38} + 500(1.03)^{37} + \dots$

(each deposit is like its own investment earning interest)

Will take way too long!

We can use "The Value of Money" (TVM) Solver on the TI Calculators:

APPS → Finance → TVM Solver

- N = Number of Payments
- I% = Interest rate (as a %)
- PV = Principal Value (0 if you are starting with nothing)
- PMT = Amount of payments (NEGATIVE if you are making payments)
- FV = Future Value
- P/Y = Payments per year (minimum = 1)
- C/Y = Compoundings per year (minimum = 1)
- PMT: **END** BEGIN

Note:

Money out of your pocket is negative.

Money going into your pocket is positive.

* **Always leave it on END** *

(unless the question specifically says that payments are made at the beginning of each month)

So for our question.....

- N = $2 \times 20 = 40$
- I = 6
- PV = 0
- PMT = -500
- FV =
- P/Y = 2
- C/Y = 2
- PMT: **END** BEGIN

Move cursor here and hit ALPHA then ENTER

When you are ready to Calculate your answer:

Move your cursor to the one you want to find and press **ALPHA** then **ENTER**

Final Value = \$37700.63

Example 2: Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits \$500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually.

a) How much money will be in the account at the end of 5 years?

$$\begin{aligned}
 N &= 2 \times 5 = 10 \\
 I &= 3.8 \\
 PV &= 0 \\
 PMT &= -500 \\
 FV &= \text{cursor here} \\
 P/Y &= 2 \\
 C/Y &= 2 \\
 PMT: & \text{END} \quad \text{BEGIN}
 \end{aligned}$$

$$FV = \$5449.90$$

b) How much of this money will be earned interest? She invested $500 \times 10 = \$5000$

$$\begin{array}{r}
 5449.90 \\
 - 5000.00 \\
 \hline
 \$449.90
 \end{array}$$

$$\$449.90 \text{ earned in Interest}$$

Example 3: Adam made a \$200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

a) What was their future value?

$$\begin{aligned}
 N &= 1 \times 5 = 5 \\
 I &= 5 \\
 PV &= 0 \\
 PMT &= -200 \\
 FV &= \text{cursor} \\
 P/Y &= 1 \\
 C/Y &= 1 \\
 PMT: & \text{END} \quad \text{BEGIN}
 \end{aligned}$$

$$FV = \$1105.13$$

b) What principal amount did Blake invest 5 years ago? $A = P(1+i)^n$

$$\begin{aligned}
 1105.13 &= P(1 + 0.05)^5 \\
 1105.13 &= P(1.276281563)
 \end{aligned}$$

ANS ANS

$$P = \$865.90$$

c) Who earned more interest? Why?

Adam

$$\text{Invested } 200 \times 5 = \$1000$$

$$\text{Interest} = \$105.13$$

Blake

$$\begin{array}{r}
 \text{Interest} = 1105.13 \\
 - 865.90 \\
 \hline
 \$ 239.23
 \end{array}$$

Blake had more money earning interest over all 5 years

Example 4: Jeremiah deposits \$750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is \$10 059.07. What annual rate of interest does Jeremiah's investment earn?

$N = 4 \times 3 = 12$
 $I = \text{CURSOR}$
 $PV = 0$
 $PMT = -750$
 $FV = 10059.07$
 $P/Y = 4$
 $C/Y = 4$
 $PMT: \text{END} \text{ BEGIN}$

$$I = 8.000019121$$

Rate of Interest
is 8%

Example 5: Celia wants to have \$300 000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

a) What regular payments must Celia make at the end of each year to meet her goal of \$300 000?

$N = 20$
 $I = 10.8$
 $PV = 0$
 $PMT = \text{CURSOR HERE}$
 $FV = 300000$
 $P/Y = 1$
 $C/Y = 1$
 $PMT: \text{END} \text{ BEGIN}$

$$PMT = -\$4781.09$$

Celia must make
payments of
\$4781.09

b) How much interest will she earn over the 20 years?

$$20 \text{ payments} \times 4781.09 = 95621.80$$

$$\text{Interest} = 300000.00 - 95621.80$$

$$= \$204378.20$$

Example 6: On Luis's 20th birthday, he started making regular \$1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually. At what age will he have more than \$18 000?

$N = \text{CURSOR HERE}$
 $I = 3.5$
 $PV = 0$
 $PMT = -1000$
 $FV = 18000$
 $P/Y = 2$
 $C/Y = 2$
 $PMT: \text{END} \text{ BEGIN}$

$$N = 15.784$$

\therefore 16 payments will make more than \$18000.

That is 8 years

so Luis will be 28
years old

Assignment: Pg. 55 #1-3, and 5-15 (odds) (Extension #18,19)

Online TVM Solver: <http://www.zenwealth.com/BusinessFinanceOnline/TVM/TVMCalculator.html>