4.2 – Introducing Permutations & Factorial Notation

Permutation: An arrangement of distinguishable objects in a definite order. (**order** matters)

Example: the objects *A* and *B* have two different permutations: *AB* and *BA*

Example 1: List all the permutations are there of the letters *A*, *B*, and *C*.

How many permutations are possible?

ABC
ACB
BAC
BCA
CAB
CBA

6 permutations are possible

Example 2: Determine the number of arrangements that six children can form while lining up to drink.

Position A	Position B	Position C	Position D	Position E	Position F
↑	↑	↑	↑	↑	↑
6 options for Position A	once one kid is in A there are 5 options for position B	2 kids lined up already so 4 options	3 options	2 options	1 option.

Permutations $= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$\boxed{= 720 \text{ permutations}}$

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ can be written as $6!$

"six factorial"

Factorial Notation: A concise representation of the product of consecutive descending natural numbers

$$n! = n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$$

Example: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Example 3: Evaluate the following

a) $10! = 10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = \boxed{3628800}$

* Can use factorial button on your calculator.
TI Calculators → $\boxed{\text{MATH}}$ ▶ ▶ ▶ $\boxed{\text{PRB}}$ 4:!

b) $\dfrac{12!}{9!3!} = \dfrac{12!}{3!9!}$

$= \dfrac{12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1)}$

Anything divided by itself = 1

$= \dfrac{12\cdot11\cdot10}{3\cdot2\cdot1} = \dfrac{1320}{6} = \boxed{220}$

Example 4: Simplify, $(n \in \mathbb{N})$ → n is a Natural Number $\{1, 2, 3, 4, 5, \ldots\}$

a) $(n+3)(n+2)!$

$= (n+3)(n+2)(n+1)(n)(n-1)(n-2)(n-3)\cdots(3)(2)(1)$

$\boxed{= (n+3)!}$

b) $\dfrac{(n+1)!}{(n-1)!} = \dfrac{(n+1)(n)(n-1)(n-2)(n-3)\cdots(3)(2)(1)}{(n-1)(n-2)(n-3)\cdots(3)(2)(1)}$ $=1$

$= (n+1)(n)$

$\boxed{= n^2 + n}$

n is an Integer $\{\ldots,-3,-2,-1,0,1,2,3\ldots\}$

Example 5: Solve $\frac{(n+4)!}{(n+2)!} = 6$, where $(n \in \mathrm{I})$

$$\frac{(n+4)(n+3)\boxed{(n+2)(n+1)\cdots(3)(2)(1)}}{\boxed{(n+2)(n+1)\cdots(3)(2)(1)}} = 6$$

$= 1$

$$(n+4)(n+3) = 6$$

	n	$+4$
n	n^2	$+4n$
$+3$	$+3n$	$+12$

$$n^2 + 3n + 4n + 12 = 6$$

$$n^2 + 7n + 12 = 6$$

$$\quad -6 \quad -6$$

$$n^2 + 7n + 6 = 0$$

$1 \times 6 = 6$

$1 + 6 = 7$

	n	$+1$
n	n^2	$1n$
$+6$	$6n$	$+6$

$$(n+6)(n+1) = 0$$

only way to get zero is if

$n = -6$ OR $n = -1$

Check Answers!

$n = -6$	$n = -1$
$\frac{(-6+4)!}{(-6+2)!} = 6$?	$\frac{(-1+4)!}{(-1+2)!} = 6$?
$\frac{(-2)!}{(-4)!} = 6$?	$\frac{3!}{1!} = 6$?
	$\frac{3 \cdot 2 \cdot 1}{1} = 6$?
	$\frac{6}{1} = 6$?

Factorials of negative numbers are undefined!

$\therefore n \neq -6$

Assignment: Pg. 243 #1 – 13, 15 (every other letter for #1, 3 – 6) Extension (#17,18)

$6 = 6$ ✓

$$\boxed{\therefore n = -1}$$