FOM 12

4.3 – Permutations When All Objects Are Distinguishable

(Different)

Example 1: You have 10 books and need to put 4 of them on a shelf.

a) How many ways are there of selecting and ordering the books?

Shelf: 10 choices, 9 choices, 8 choices, 7 choices

$10! = 10 \cdot 9 \cdot 8 \cdot 7 (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$

so need to $\div$ by $6!$

$10 \cdot 9 \cdot 8 \cdot 7 = \frac{10!}{6!} = \boxed{5040 \text{ ways}}$ $\rightarrow \frac{(10 \text{ books})!}{(10 - 4 \text{ on shelf})!}$

b) How would the factorial expression look for 3 books being selected from the 10?

Shelf: $\frac{10 \cdot 9 \cdot 8 \cdot (7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{\div (7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = \frac{10!}{7!} \rightarrow \frac{(10 \text{ books})!}{(10 - 3 \text{ on shelf})!}$

$\boxed{= 720 \text{ ways}}$

c) What if there were 100 books and we can only select 20?

$\frac{(100 \text{ books})!}{(100 - 20 \text{ on shelf})!} = \frac{100!}{80!} = \boxed{\text{Really Big \#!}}$

d) What if I have n books and I can select r of them?

$\frac{n!}{(n-r)!}$

Notation: $_nP_r$ "from n Permute r"

or more commonly "from n Pick r"

Permutation Formula. $\boxed{_nP_r = \frac{n!}{(n-r)!}}$

Example 2: Matt has downloaded 10 new songs from an online music store. He wants to create a playlist using 6 of these songs arranged in any order. How many different 6-song playlists can be created from his new downloaded songs?

$_nP_r = \frac{n!}{(n-r)!}$

"from n choose r"

"from 10 choose 6" $_{10}P_6 = \frac{10!}{(10-6)!} = \frac{10!}{4!}$

* simply put on calculator or simplify first.

$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \boxed{4 \cdot 3 \cdot 2 \cdot 1}}{\boxed{4 \cdot 3 \cdot 2 \cdot 1}}$

$\boxed{= 151200 \text{ playlists}}$

And For Your Information: $0! = 1$ (proof is on page 249)

Example 3: Tania needs to create a password for a social networking website she registered with. The password can use any digits from 0 to 9 and /or any letters of the alphabet. The password is case sensitive, so she can use both lower – and upper-case letters. A password must be at least 5 characters to a maximum of 7 characters, and each character can be used only once in the password. How many different passwords are possible?

Number of characters we can use:

26 lower-case + 26 upper-case + 10 numbers

= 62 characters

Case 1: 5 characters $\quad {}_{62}P_5 = \frac{62!}{(62-5)!} = \frac{62!}{57!} = 776520240$

Case 2: 6 characters $\quad {}_{62}P_6 = \frac{62!}{(62-6)!} = \frac{62!}{56!} = 4.426165 \times 10^{10}$

Case 3: 7 characters $= {}_{62}P_7 = \frac{62!}{(62-7)!} = \frac{62!}{55!} = 2.479 \times 10^{12}$

$= 2.524 \times 10^{12}$

$\boxed{= 2{,}524{,}000{,}000{,}000 \text{ different passwords}}$

Example 4: A social insurance number (SIN) in Canada consists of a nine-digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SIN's can be created if each digit can be repeated? How does this compare with the number of SIN's that can be created if no repetition is allowed?

Repetition Allowed: $\underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}} \cdot \underset{\#'s}{\underline{10}}$

= 1,000,000,000 different numbers

Repetition Not Allowed

$${}_{10}P_9 = \frac{10!}{(10-9)!} = \frac{10!}{1!}$$

$$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$\boxed{= 3628800 \text{ different numbers}}$

* Can use nPr button on calculator to make calculations easier

Example 5: At a used car lot, seven different car models are to be parked close to the street for easy viewing.

a) The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

Use a diagram:

$\boxed{3}\ \boxed{4}\ \boxed{3}\ \boxed{2}\ \boxed{2}\ \boxed{1}\ \boxed{1}$

3 red cars to choose

4 cars to choose from

3 cars

2 red cars

1 red car left to choose

$= 3 \cdot 4 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$

$\boxed{= 144 \text{ ways to park the cars}}$

Use Permutations

$= {}_3P_3 \times {}_4P_4$

order red cars

order other cars

$= 6 \times 24$

$\boxed{= 144 \text{ ways}}$

b) The three red cars must be parked side by side. How many ways can the seven cars be parked?

think of red cars as one item

$\underbrace{\boxed{{}_3P_3}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}}_{{}_5P_5}$

${}_3P_3 \times {}_5P_5$

$= 6 \times 120$

$= 720$

$\boxed{\text{the cars can be ordered in 720 ways}}$

Assignment: Pg. 255 #1 – 4 (ALL), #5 – 13 (ODDS), #15, 16 Extension (#17,19-21)