Example 1: You have 3 red marbles, 5 blue marbles, and 4 green marbles. ~~the same~~
How many **distinct** ways can you line them up?

● ● ● ● ● ● ● ● ● ● ● ●

If I change these 2 marble the order is still the same.

How many ways can I arrange each colour?

3 reds = 3·2·1 = 6 ways

5 blues = 5·4·3·2·1 = 120 ways

4 greens = 4·3·2·1 = 24 ways

12 marbles in total can be ordered
in 12! = 479 001 600 ways.

ways = $\dfrac{12!}{3!\,5!\,4!}$ ← get rid of repeats = 27720 ways

Example 2: a) How many distinct ways can the letters of the word CANADA be arranged?

$\dfrac{(\text{total objects})!}{\text{factorials of each repeated item}}$ $\dfrac{6!}{3!}$ = 120 ways

3 A's → 3!

b) How many ways can the letters be arranged if the first letter must be N and the last letter must be C?

~~C~~AN~~A~~DA
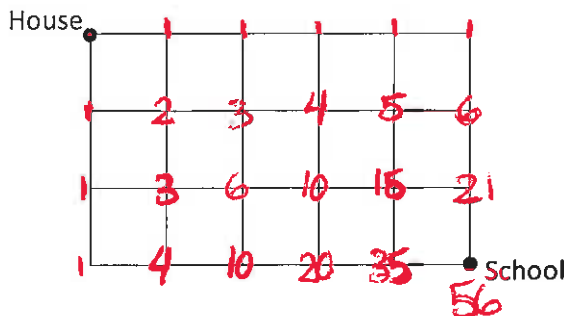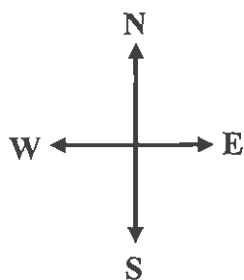
so really need to order 4 objects
(3 of which are repeated)
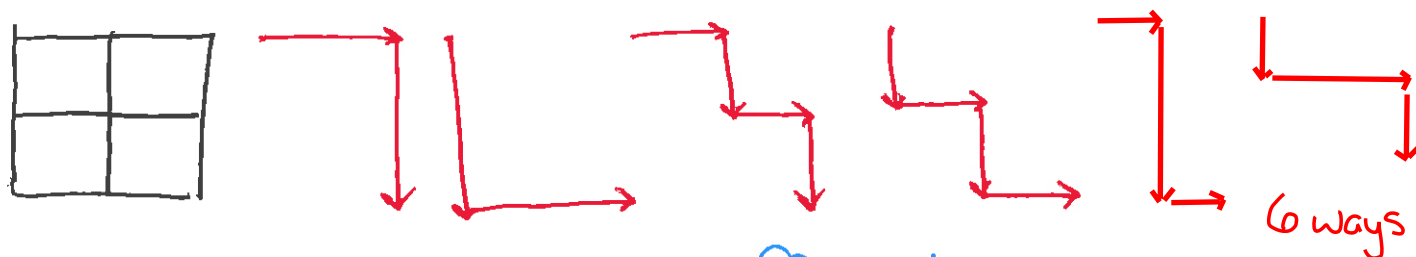
$\dfrac{4!}{3!}$ = 4 ways

Example 3: Julie's home is three blocks north and five blocks west of her school.
How many routes can Julie take from home to school if she always travels either south or east?

N
↑
W ← + → E
↓
S

House
2 3 4 5 6
3 6 10 15 21
4 10 20 35 • School
56

↓ →
(not ↑ or ←)

✱ there is a pattern! add along lines.

2 ways

3 ways

6 ways

∴ 56 different routes.

OR

No matter which route she takes there are 3 steps south & 5 steps East
(8 steps altogether)
so ordering 8 objects, but 3 south & 5 east are repeated.

$$\frac{8!}{3!\,5!} = \frac{40320}{720} = 56$$

there are 56 different routes

Assignment: Pg. 266 #1 – 11, 13, 16, 17   Extension (#18, 21)