FOM 12

4.6 – Combinations

Combination: A grouping of objects where **order does not matter.**

Example: The two objects *A* and *B* have one combination because *AB* is the same as *BA*.

Permutation: An arrangement of distinguishable objects in a definite order. (**order matters**)

Example: the objects *A* and *B* have two different permutations: *AB* and *BA*

Example 1: There is a group of five friends. A B C D E

a) Two need to go get pizza. How many different groups are possible?

AB BC CD DE
AC BD CE
AD BE
AE

BA is the same as AB

CB = BC
CA = AC
DA = AD
DB = BD
DC = CD

→ $\boxed{\text{10 different groups are possible}}$

Instead of listing options
For a Permutation (where order doesn't matter)

$$_5P_2 = \frac{5!}{(5-2)!\,2!} \leftarrow \text{this divides out the identical groups}$$

$$= \frac{5!}{3!\,2!} = 10$$

b) What if three decide to go get pizza? How many different groups are possible?

Permutation: $_5P_3 = \frac{5!}{(5-3)!}$

but $\underbrace{ABC = ACB = BAC = BCA = CAB = CBA}$

6 of permutations are actually the same combination.

when we had groups of 2 we divided by 2 (2!)
When we have groups of 3 we divide by 6 (3!)

So $\frac{5!}{(5-3)!(3!)} = \frac{5!}{2!\,3!} =$ $\boxed{\text{10 different groups of friends}}$

"from n Choose r"

AB=BA

Combinations (Order doesn't matter) $\binom{n}{r}$ or $\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}$

where $0 \le r \le n$

AB & BA are different

Permutations (Order matters) $\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$ "n Pick r"

Example 2: A restaurant serves 10 flavours of ice cream. Danielle has ordered a large sundae with three scoops of ice cream. How many different ice cream combinations does Danielle have to choose from, if she wants each scoop to be a different flavour? (Order doesn't matter)

$${}_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10!}{3!\,7!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{3!\,\cancel{7!}}$$

$$= \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = \frac{720}{6} = \boxed{120 \text{ combinations}}$$

Example 3: Tanya is the coach of a Pole Push team that consists of nine players: five male and four female. In each competition, teams of four compete against each other to push their competitors out of a circle. The team that is successful wins.

a) How many different four-person teams does Tanya have to choose from for an all-male competition?

$${}_5C_4 = \frac{5!}{4!(5-4)!} = \frac{5!}{4!\,1!} = \frac{120}{24\cdot 1} = 5$$

$\boxed{\text{5 different teams}}$

b) How many different four-person teams does Tanya have to choose from, with two makes and two females, for a mixed competition?

Men $${}_5C_2 = \frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\,2!} = \frac{120}{12} = 10$$

Women $${}_4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{24}{4} = 6$$

$$\# \text{ Teams} = 10 \times 6 = 60$$

Multiply because it is one team, not 2 different cases.

$\boxed{\text{there are 16 different mixed teams possible}}$

* Can use nCr button on your calculator

Example 4: A planning committee is to be formed for a school-wide Earth Day program. There are 13 volunteers: 8 teachers and 5 students. How many ways can the principal choose a 4-person committee that has at least 1 teacher?

Must calculate each case separately:

1 teacher 3 students $\quad {}_8C_1 \times {}_5C_3 = \frac{8!}{1!7!} \times \frac{5!}{2!3!} = 8 \times 10 = 80$

2 teachers 2 students $\quad {}_8C_2 \times {}_5C_2 = \frac{8!}{2!6!} \times \frac{5!}{2!3!} = 28 \times 10 = 280$

3 teachers 1 student $\quad {}_8C_3 \times {}_5C_1 = \frac{8!}{3!5!} \times \frac{5!}{1!4!} = 56 \times 5 = 280$

4 teachers 0 students $\quad {}_8C_4 \times {}_5C_0 = \frac{8!}{4!4!} \times \frac{5!}{0!5!} = 70 \times 1 = 70$

Add all possibilities

$$80 + 280 + 280 + 70 = 710$$

710 different committees are possible

Example 5: a) How many poker hands exist? (5 card hand)

$${}_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} = \boxed{2598960}$$

b) How many poker hands have a full house? (3 of a kind and 2 of a kind)

Fundamental Counting Principle: "a" ways to perform one task & "b" ways to perform another means a·b ways to perform both.

3 of kind		2 of kind
pick suits ${}_4C_3$ 13 "values" to choose from	$\times$	pick suits ${}_4C_2$ Now only 12 "values" to choose from

$$= (13 \cdot {}_4C_3)(12 \cdot {}_4C_2)$$

$$= 13(4)(12)(6) = 3744$$

different full-house poker hands

Assignment: Pg. 280 #1 – 4, 5, 7, 11, 13, 14 – 16 Extension (#21, 22)