FOM 12

4.7 – Solving Counting Problems

When solving problems, ask yourself:

"Does Order Matter?"

AB and BA are different

YES

Permutations

$${}_nP_r = \frac{n!}{(n-r)!}$$

NO

Combinations

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

AB is same as BA

Example 1: A piano teacher and her students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of nine chairs for this pose?

Order Matters ⇒ Permutations

Order Boys	Order Girls	Order Teachers
3!	5!	1!

order Blocks 3!

$3! \times 5! \times 1! \times 3!$

$= 6 \times 120 \times 1 \times 6$

$= 4320$ ways to pose for picture

Example2: Combination problems are common in computer science. Suppose there is a set of 10 different data items represented by ($a, b, c, d, e, f, g, h, i, j$) to be placed into four different memory cells in a computer. Only 3 data items are to be placed into the first cell, 4 data items in the second cell, 2 data items in the third cell, and 1 data item in the last cell. How many ways can the 10 data items be placed in the four memory cells?

order doesn't matter. (combination)

Cell 1 (10 data items) | Cell 2 (7 items left) | Cell 3 (3 items left) | Cell 4 (1 item left)

$_{10}C_3 \times {}_7C_4 \times {}_3C_2 \times {}_1C_1$

$120 \times 35 \times 3 \times 1$

= 12600 ways to place data

Example 3: How many different five-card hands that contain **at most** one black card can be dealt to one person from a standard deck of playing cards?

Order doesn't matter (combinations)
(add parts because they are different cases)

0 black 5 Red $= {}_{26}C_0 \times {}_{26}C_5 = 1 \times 65780 = 65780$

1 Black 4 Red $= {}_{26}C_1 \times {}_{26}C_4 = 26 \times 14950 = 388700$

$65780 + 388700$

= 454480 different hands

Assignment: Pg. 288 #1 – 3 (ALL), 5 – 15 (ODDS)   Extension (#18,19)