FOM 12

5.3 – Probabilities Using Counting Methods

Example 1: Four people (A, B, C, D) are going to line up. If the line-up is randomly generated, what is the probability that person D is first?

How many possibilities are there? (Permutation)

$4! = 24$ possibilities

How many possibilities with D first $\underset{D}{1} \cdot 3 \cdot 2 \cdot 1$

$1 \times 3 \times 2 \times 1 = 6$

6 possibilities

$P(\text{D first}) = \frac{6}{24} = \frac{1}{4} = 25\%$

Example 2: Ten names are written on slips of paper and placed in a hat. Roger will win a prize if his name is drawn. Three names are to be drawn out at random. What is the probability that Roger wins?

How many ways can we choose 3 slips of paper from 10? ${}_{10}C_3 = 120$

How many ways can we choose 3 slips of paper which include Roger?

${}_1C_1 \rightarrow$ choose Roger

${}_9C_2 \rightarrow$ out of 9 remaining choose 2

$1 \times {}_9C_2 = 1 \times 36 = 36$

$P(\text{Roger wins}) = \frac{36}{120} = \frac{3}{10} = 30\%$

Example 3: Beau hosts a morning show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with scrabble tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

How many ways can SASKATCHEWAN be permuted?

$$\frac{12!}{2!3!} = 39916800$$

How many ways actually spell SASKATCHEWAN?

$$P(\text{winning car}) = \frac{1}{39916800}$$

Example 4: There are 18 bikes in Marnie's spinning class. The bikes are arranged in 3 rows, with 6 bikes in each row. Allison, Brett, Carol, Doug, Erica, and Franco each call the gym to reserve a bike. They hope to be in the same row, but they cannot request a specific bike. Determine the probability that all 6 friends will be in the same row with Allison and Franco at either end.

Assume: Bikes assigned randomly & all bikes are used.

Number of ways to assign all 18 people: $18!$ or $({}_{18}P_{18})$

Number of ways to assign 6 friends in 1 row with Allison Franco at either end: ${}_2P_2 \times {}_4P_4$

Number of ways to assign 6 friends in 1 row as above add sit all other 12 people ${}_2P_2 \times {}_4P_4 \times {}_{12}P_{12}$

But friends could be in any of 3 Rows

so $3({}_2P_2)({}_4P_4)({}_{12}P_{12})$

see next page!

Assignment: Pg. 321 #1 – 5, 7, 8, 11, 12, 13, 14 Extension (#19)

$P(\text{friends in Row with A \& F on ends}) =$

$$\frac{3({}_2P_2)({}_4P_4)({}_{12}P_{12})}{{}_{18}P_{18}}$$

$$= \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18!}$$

$$= \frac{3 \cdot 2! \cdot 4! \cdot \cancel{12!}}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12!}}$$

$$= \frac{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}$$

$$= \frac{144}{13366080}$$

$$\boxed{= \frac{1}{92820}}$$

Side Note:

${}_4P_4 = \frac{4!}{(4-4)!}$

$= \frac{4!}{0!}$

$= \frac{4!}{1}$

$= 4!$

So....