7.1 – Exponential Functions

Exponential Functions are based on the concept of repeated multiplication.

For example folding a sheet of paper:

Number of Folds (n)	Number of Layers (l)
0	1
1	2
2	4
3	8
4	16
5	32

×2 ×2 ×2 ×2 ×2

×2 for each step

$l = 2^n$ ← Variable is in the Exponent

↑ Multiplier "BASE"

How many layers would there be with 10 folds?

$l = 2^n = 2^{10} = 1024$

How many layers would there be with 12 folds?

$l = 2^{12} = 4096$

(exponential functions increase quickly → only 2 more folds & already 4 times greater)

Exponential Function: A function of the form $y = a(b)^x$ where $a \neq 0, b > 0, and\ b \neq 1$.
(variable is in the exponent)

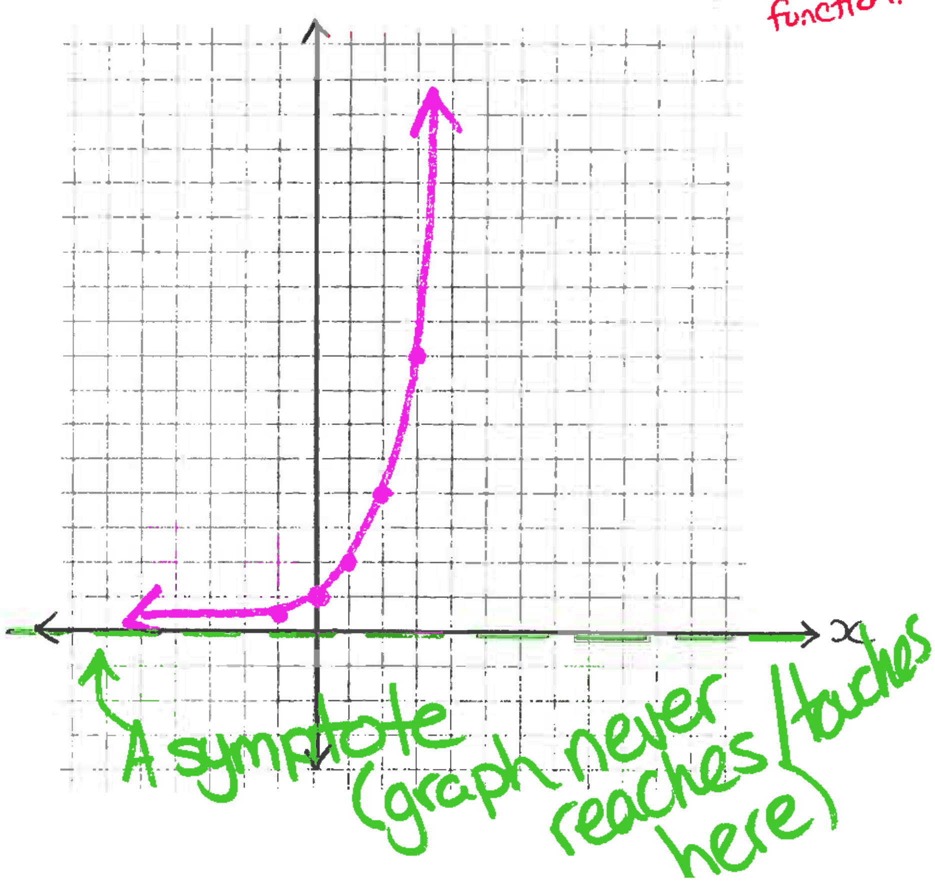
* base (b) cannot be negative.
* If b=1 it would be a constant function.

Graphs of Exponential Functions:

Example 1: $y = 2^x$

x is independent variable

x	y	
−1	1/2	(2^{-1})
0	1	(2^0)
1	2	(2^1)
2	4	(2^2)
3	8	(2^3)

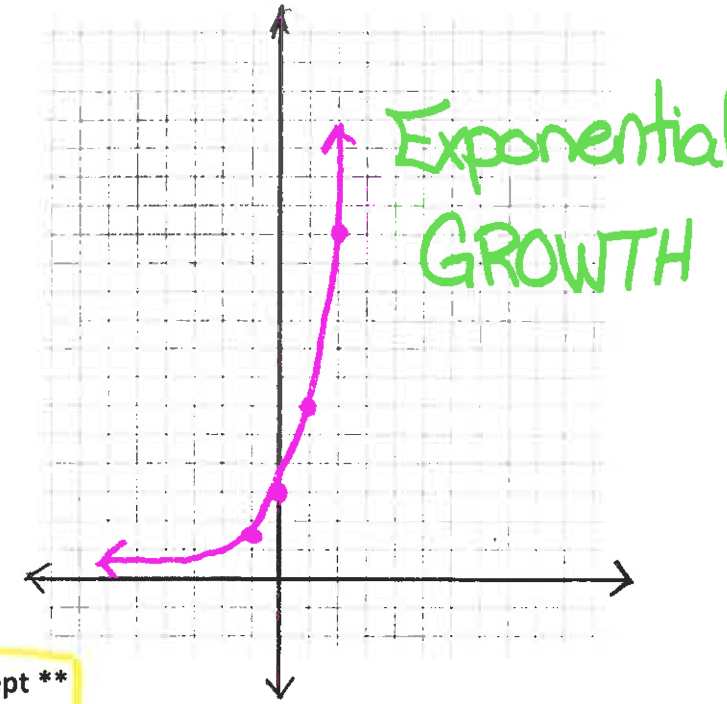


Observations:

→ Exponential growth starts slow but eventually increases very quickly
→ Exponential functions have a horizontal asymptote (a line for which the graph will get closer and closer to but never actually touch)
→ No x-intercepts
→ Only one y-intercept
→ End Behaviour: QII to QI
→ Domain: $x \in \mathbb{R}$
→ Range: $y > 0$

Example 2: $y = 3(2)^x$

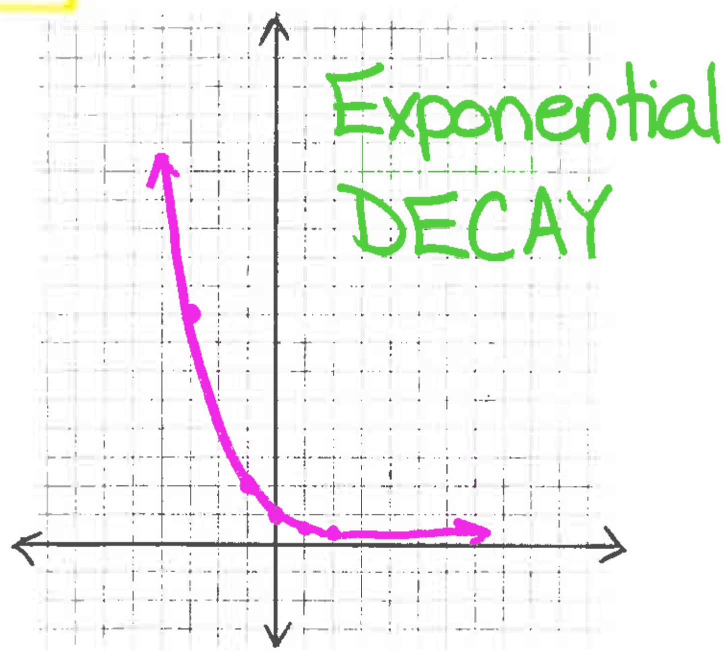
x	y
-1	$3(2)^{-1} = \frac{3}{2} = 1.5$
0	$3(2)^0 = 3(1) = 3$
1	$3(2)^1 = 3(2) = 6$
2	$3(2)^2 = 3(4) = 12$



**** The number in front of the base is the y-intercept ****

Example 3: $y = \left(\frac{1}{2}\right)^x$

x	y
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = 0.5$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$
-3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$



**** When the base is $0 < b < 1$ we get Exponential Decay and $b > 1$ is Exponential Growth ****

Assignment: Pg. 439 #1 – 3