FOM 12

7.4 – Characteristics of Logarithmic Functions with Base 10 and Base e

A Logarithm is an Exponent written in a different way

$10^2 = 100 \longrightarrow \log_{10}(100) = 2$

$10^3 = 1000 \longrightarrow \log_{10}(1000) = 3$ → The Exponents

$10^1 = 10 \longrightarrow \log_{10}(10) = 1$ →

Example 1: What is the value of $\log_{10}(20)$?

(use LOG on your calculator)

Any time it just says "log" that implies base 10 (just like $\sqrt{\ }$ implies $\sqrt[2]{\ }$)

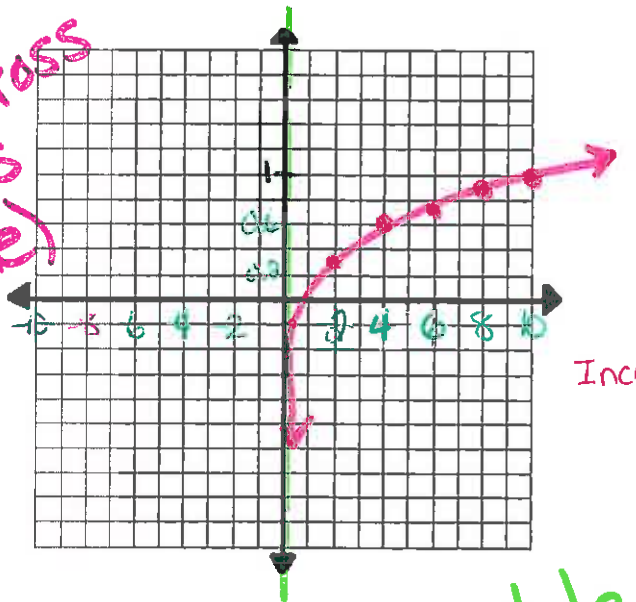
LOG (20) = 1.301029996

$\log 20 = 1.3$

Example 2: Sketch a graph of the function $y = \log_{10}(x)$

x	y
0	$\log 0$ = undefined
2	$\log 2 = 0.30$
4	$\log 4 = 0.60$
6	$\log 6 = 0.78$
8	$\log 8 = 0.90$
10	$\log 10 = 1$
0.1	$\log 0.1 = -1$

← won't touch/cross y-axis (asymptote)



Increasing.

↑ Asymptote

Domain: $x > 0$

Range: $y \in \mathbb{R}$

Introducing "e": e is a constant number named e in honour of Euler who proved that it was irrational. With the possible exception of π, e is the most important constant in mathematics since it appears in myriad mathematical contexts involving limits and derivatives. The numerical value of e is

$e = 2.71828182845904523536028747135266249775 7 \ldots$

Logarithms with a base of e are called Natural Logarithms.

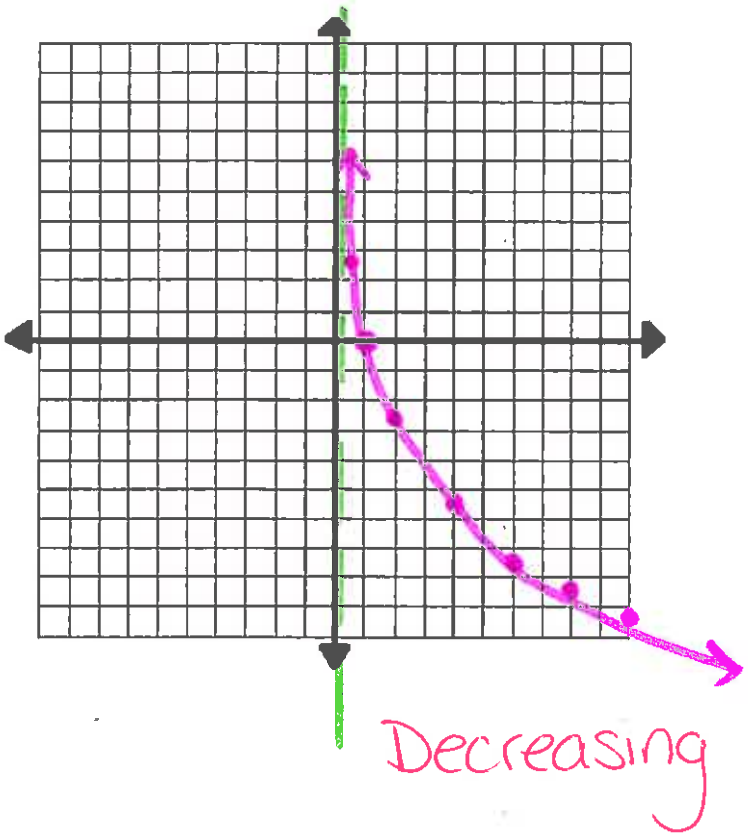
$y = \log_e(x)$ is equivalent to $y = \ln(x)$

Example 3: Sketch a graph of the function $y = -4\ln(x)$

0	$-4\ln(0)$ undefined
2	$-4 \times \ln(2) = -2.77$
4	$-4 \times \ln(4) = -5.55$
6	$-4 \times \ln(6) = -7.17$
8	$-4 \times \ln(8) = -8.32$
10	$-4 \times \ln(10) = -9.21$
1	$-4 \times \ln(1) = 0$
0.5	$-4 \times \ln(0.5) = 2.77$

Asymptote

see what happens close to zero



Determine:

the x-intercept: (1,0)

number of y-intercepts: zero

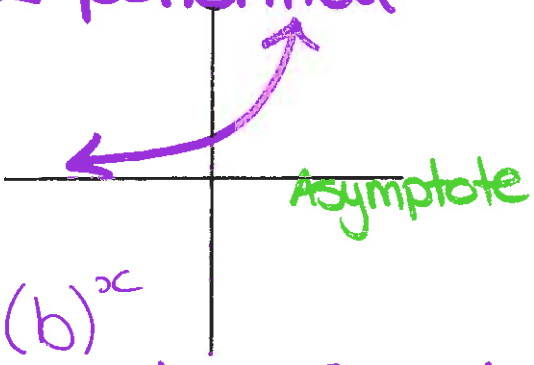
end behaviour: QI to QIV

domain and range: Domain: $x > 0$

Range: $y \in \mathbb{R}$

Comparing Exponential Functions & Logarithmic Functions

Exponential



$y = a(b)^x$

$b > 1$ = Increasing

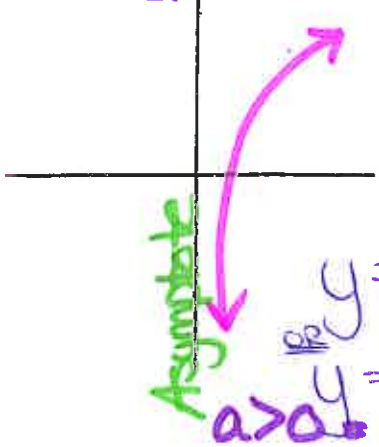
$0 < b < 1$ = Decreasing

y-intercept = a

Domain: $x \in \mathbb{R}$

Range: $y > 0$

LOGARITHMIC



$y = a\log(x)$ or $y = a\ln(x)$

$a > 0$ = Increasing

$a < 0$ = Decreasing

x-intercept = 1

Domain: $x > 0$

Range: $y \in \mathbb{R}$

Example 4: Which function matches each graph below?

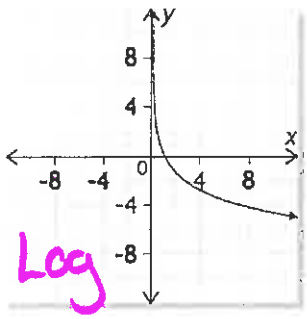
Exp ↑ $y = 5(2)^x$ y-int ↗

Exp ↓ $y = 2(0.1)^x$ y-int ↗

Log ↑ $y = 6\log x$ x-int = 1

Log ↓ $y = -2\ln x$ x-int = 1

a)

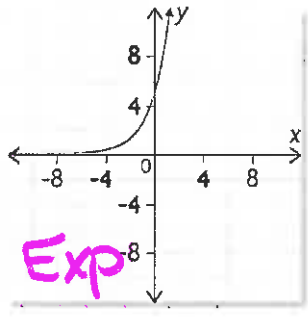


Log

Decreasing

$\therefore y = -2\ln(x)$

b)

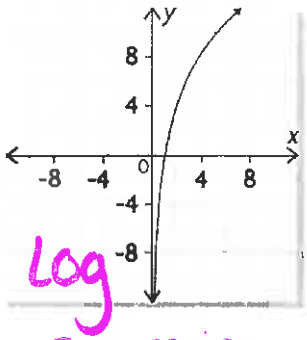


Exp

Increasing

$y = 5(2)^x$

c)

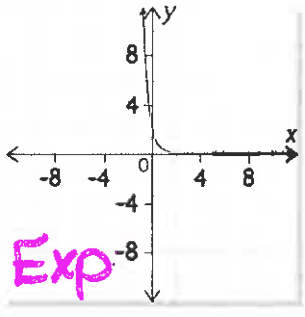


Log

Increasing

$y = 6\log(x)$

d)



Exp

Decreasing

$y = 2(0.1)^x$

Assignment: Pg. 482 #1, 2, 4, 5, 6, 8, 10, 12 (Extension #14, 15)