**Foundation of Mathematics and Pre-Calculus 10**

**Chapter 2**

**Polynomials**

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**2.1 Classifying Polynomials**

Term: a number or a product of a number with one or more variables

which can be raised to a power.

1

2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Term | 5y | -2a3 |  x2yz4  | x | 10 |  |
| Coefficients |  |  |  |  |  |  |
| Variables |  |  |  |  |  |  |

Polynomial: a term or sum of terms, in which all variables have whole number exponents,

and in which variables appear only in the numerator.

|  |  |
| --- | --- |
| Polynomials | Non-Polynomials |
|  |  |
|  |  |
|  |  |

Classifying Polynomials

|  |  |  |  |
| --- | --- | --- | --- |
| Monomial | Binomial | Trinomial | Polynomial |
| A polynomial with one term | A polynomial with two terms | A polynomial with three terms | General term for expressions with more than three terms (can also be used to describe monomial, binomial, or trinomial) |
|  |  |  |  |
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Degree of a Polynomial: the sum of the exponents of the variables in that term.

 The term with the highest degree.

|  |  |  |
| --- | --- | --- |
| Example | The term of highest degree | Degree |
|  |  |  |
|  |  |  |
|  |  |  |

Leading Term: the term with the highest degree.

Like Terms: either constant terms, or terms that contain the same variable(s) to the same power

 Combine the like terms:

 1. 2. 3.

 4. 5. 6.

Evaluating Polynomial

 When a constant is substituted for a variable in a polynomial,

the polynomial is evaluated for the constant

 Examples:

 1. 2. 3.

Try these:

Complete the table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Expressions | Classification | Coefficient/s | Variable/s | Degree of each term | Degree of polynomial | Leading term |
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Multiplying Monomial

 Examples:

 1. (3a)(2a2) 2. 3.

Multiplying a Polynomial by Binomial

 Examples:

 1. 5w(3w3 – 4w + 2 2. 3.

 4. 5. 6.

**2.2 Multiplying Polynomials**

Binomial by Binomial

|  |
| --- |
| Methods |
| Distributive |
| (a + 1) ( 3a + 4) | (2a – 3) (a + 2) |
| Vertical |
| 2a + 5  x 3a + 4  | 2a – 3  x a + 2 |
| Rectangle |
| (2a + 5) ( 3a + 4)

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |

 | (2a – 3) (a + 2)

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |

 |
| FOIL (First, Outside, Inside, Last) |
| (2a + 5) ( 3a + 4) | (2a – 3) (a + 2) |
| SMILE |
| (2a + 5) ( 3a + 4) | (2a – 3) (a + 2) |

**General Rules**

 Square of a Binomial

 (a + b)2 = a2 + 2ab + b2

 (a - b)2 = a2 - 2ab + b2

Short Cut Method

(a + 3)2

Step 1: Multiply the 1st term by itself (a)(a) = a2

Step 2: Double the product of the 1st and 2nd term 2(a)(3) = 6a

Step 3: Multiply the 2nd term by itself (3)(3) = 9

Therefore the answer is = a2 + 6a + 9

Try these:

1. 2. 3.

Product of Sum and Difference

 (a – b) (a + b) = a2 – b2

Short Cut Method

 (a – 3)(a + 3)

Step 1: Multiply the 1st terms (a)(a) = a2

Step 2: Multiply the 2nd terms (-3)(3) = -9

Therefore the answer is = a2 – 9

Try these:

1. 2. 3.

Binomial by Trinomial

1. 2. 3.

Trinomial by Trinomial

1. 2. 3.

**2.3 Removing Common Factors**

Review: GCF

If every term is a polynomial has several factors, and if every term has at least one factor that is the same, then that factor is called a **common factor**.

Example:

Set A (factor)

1. 5x + 10 2. 3x2 – 6 3. 12x4 – 8x3 + 4x2

Set B (factor)

1. x(x + 1) + 3(x + 1) 2. m(t – h) -a(t – h) 3. b(a + c) d(a + c)

Set C (factor by grouping)

1. x3 + x2 + 3x + 3 2. 2x3 – 6x2 + x – 3 3. a2 + ab – 2a – 2b

**2.4 Factoring x2 + bx + c**

Set A: x2 +bx + c

 1. x2 + 5x + 6 2. x2 + 7x + 12 3. x2 + 8xy + 15

 4. x2 + 10x + 16 5. x2 + 15x + 36 6. x2 + 12xy + 32

Set B: x2 – bx + c

 1. x2 – 3x + 2 2. x2 – 6x + 8 3. x2 – 9x + 14

 4. x2 – 5x + 6 5. x2 – 11x + 28 6. x2 – 10x + 24

Set C: x2 – bx – c

 1. x2 – x – 2 2. x2 – x – 30 3. x2 – x – 56

 4. x2 – 2x – 15 5. x2 – 6x – 16 6. x2 – 4x – 45

Set D: x2 + bx – c

 1. x2 + x – 6 2. x2 + x – 20 3 x2 + x – 72

 4. x2 + 3x – 4 5. x2 + 4x – 12 6. x2 + 9x – 22

Set E: -x2 bx c

 1. -x2 + 3x – 2 2. -x2 + 5x – 6 3. -x2 + 3x + 18

 4. -x2 + x + 6 5. -x2 – 3x + 10 6. -x2 – 5x + 36

More examples:

 1. x2 + 8 – 6x 2. x2 + 21 + 10x 3. x2 + 8xy + 15y2

 4. x2 + 14xy + 40y2 5. 5x2 + 35x + 60 6. 4x2 + 12x – 16

 7. -3x4 – 18x3 – 27x2 8. -2x2 + 10x – 12

Factor Completely

2

9

1. x2 – x + 2. x6n – 3x3n + 2

3. x3(2a + 5) + 9(2a + 5) – 10(2a + 5)

4. (3a – b)y2 - 13(3a – b)y + 40(3a – b)

5. 3a2m + 8ambn – 3b2n

**2.5 Factoring ax2 + bx + c**

Using the **AC – Method**

Steps: 1. Factor out a common factor, if it exists

 2. Change ax2 + bx + c 🡺 ax2 + *b*x + *ac*

multiply

 3. Find two integers whose product is *ac*, and whose sum is *b*

 4. Divide each constant factor by a, and simplify

 5. If a fraction remains, the denominator becomes the coefficient

 of the x or 1st term in that binomial

Examples

 1. Factor 2x2 + 7x – 4 Step

 Solution: 2x2 + 7x – 4 🡺 2x2 + 7x – 8 (2)

 (2)(-4)= -8

 2x2 + 7x – 8 (3)

 (x + 8) (x – 1)

 (x + 8) (x – 1) (4)

 2 2

 (x + 4) (2x – 1) (5)

 2. Factor 12x2 – 5x – 2

 3. 8x2 + 18x + 9 4. 8x2 – 14x + 3

5. -8x3 + 10x2 + 12x 6. – 36x2 – 96xy – 64y2

7. 9 – 10x2 + x4 8. 10x2m – 4xmyn – 6y2n

**2.6 Special Factors**

Difference of Squares Factoring the Difference of Squares

 a2 – b2 = (a + b)(a – b)

 Examples

 1. x2 – 9 2. x2 – 16 3. x4 – 4y2

 4. 9x2 – 4 5. 81m6 – 25n12 6. 4 – (x + 2)2

Perfect Square Trinomial Factoring Perfect Square Trinomials

 a2 + 2ab + b2 = (a + b)2

 a2 - 2ab + b2 = (a - b)2

 Examples

 1. x2 + 4x + 4 2. x2 + 8x + 16 3. x2 – 6x + 9

 4. 4x2 + 12x + 9 5. 3a2 – 30a + 75 6. – 9d2 – 24bd – d2