

Math 9

Tripp

Name: _____ **KEY** _____

Chapter 2 – Exponents

Test Date: _____

To do:

2.1 – Exponents

- Complete Notes

2.2 – BEDMAS

- Complete Notes

2.3 – Square Roots of Perfect Squares

- Complete Notes
- Quiz 1

2.4 – Square Roots of Whole Numbers

- Complete Notes

2.5 – Pythagorean Theorem

- Complete Notes

2.6 – Cubes and Cube Roots

- Complete Notes

2.7/2.8 – Exponent Laws

- Complete Notes
- Quiz 2

Assignments

- Chapter Assignment

Write Unit Test

An exponent is a quantity representing the **power** to which a given number or expression is to be **raised**. Expressed as:

$$\begin{array}{l} 2 \leftarrow \text{exponent} \\ 3 \leftarrow \text{base} \end{array} \quad | \quad \begin{array}{l} 3^2 = 3 \circ 3 \\ 3^3 = 3 \circ 3 \circ 3 \\ 3^4 = 3 \circ 3 \circ 3 \circ 3 \end{array}$$

Examples:

1. Expand and evaluate:

a. $6^3 = 6 \times 6 \times 6 = 216$

b. $9^7 = 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 4,782,969$

A **square number/perfect square** is the result of when a number is **multiplied** by itself. For

example:

$$8^2 = 8 \times 8 = 64$$

Any **whole** number multiplied by itself will result in a **square number/perfect square**.

Perfect Squares:

Number	Number Squared	Perfect Square
1	1^2	1
2	2^2	4
3	3^2	9
4	4^2	16
5	5^2	25
6	6^2	36
7	7^2	49
8	8^2	64
9	9^2	81
10	10^2	100
11	11^2	121
12	12^2	144
13	13^2	169
14	14^2	196
15	15^2	225

A special order of operations is to be done when there are several operations needed to simplify an expression.

B—brackets

E—exponents

D—division

M—multiplication

A—addition

S—subtraction

Examples:

1. $(5 - 3)^2 + 18 \div 2$

$$= (2)^2 + 18 \div 2$$

$$= 4 + 18 \div 2$$

$$= 4 + 9$$

$$= 13$$

2. $\frac{4^3+6}{5 \times 2}$

$$= \frac{64 + 6}{5 \times 2}$$

$$= \frac{70}{10}$$

$$= 7$$

Finding the square root of a number is the **opposite** of a number squared. For example:

$$5^2 = 25$$

$$\sqrt{25} = 5$$

You can use prime factorization to find the square root of a perfect square. For example:

$$\begin{aligned}\sqrt{225} &= \sqrt{3 \times 3 \times 5 \times 5} \\ &= \sqrt{3^2 \times 5^2} \\ &= 3 \times 5 \\ &= 15\end{aligned}$$

Another way to find the square root of a number is to find all the **factors** of the number. Factors are all the numbers that **divide** into it. For example:

Factors of 64: 1, 2, 4, 8, 16, 32, 64

$$1 \times 64, 2 \times 32, 4 \times 16, 8 \times 8$$

Whenever a number has **two “copies” of a factor**, that factor is the square root of the number.

So, a perfect square will have an **odd** number of factors. For example:

Factors of 64: 1, 2, 4, 8, 16, 32, 64 → 7 factors

Examples:

1. $\sqrt{81} = 9$

$$\sqrt{49} = 7$$

$$\sqrt{400} = 20$$

2. Solve for x

$$x^2 = 36$$

$$x = 6$$

$$x^2 = 25$$

$$x = 5$$

$$x^2 = 9$$

$$x = 3$$

3. Use prime factorization and factors to evaluate.

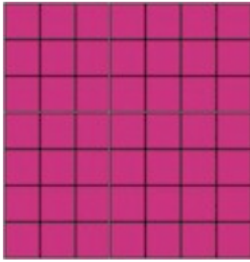
$$\sqrt{324} = \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$$

$$= \sqrt{2^2 \times 3^4}$$

$$= 2 \times 3^2$$

$$= 18$$

The square root of a number can also be determined by **finding the side length of a square** whose area is equal to that number. For example:



$$\text{Area} = 49 \text{ units}^2 = \text{side length} \times \text{side length}$$

$$\sqrt{49} = \sqrt{\text{side length}}$$

$$7 \text{ units} = \text{side length}$$

We can approximate the square root of a **whole number** that is not a perfect square by determining what **two** perfect squares it is **between**. We use the perfect squares as benchmarks. For example:

The square root of 39 must lie between the square root of 36 and the square root of 49.

Knowing this, I know that the square root of 39 is somewhere between 6 (the square root of 36) and 7 (the square root of 49).

Once you have an approximate value you can use a **number line** to be more accurate. (Use a number line)



A more accurate way of finding square roots of non-perfect numbers is to use a **calculator**. It is important to understand how this tool works in order to correctly input the calculation to get the correct outcome! For example:

$$\sqrt{13} = 3.605551275463989293119221267470495946251296573845246212\dots$$

This answer—even with all of the decimal places—is rounded!

Examples:

1. Use your calculator to evaluate. Round your answer to 2 decimal places.

$$\sqrt{88} \approx 9.38$$

$$\sqrt{32} \approx 5.66$$

2. Approximate:

$$\sqrt{41}$$

Between 6 and 7

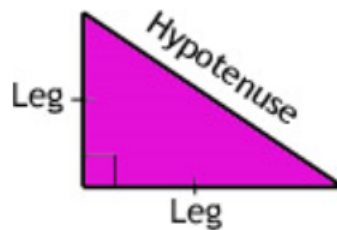
$$\sqrt{13}$$

Between 3 and 4

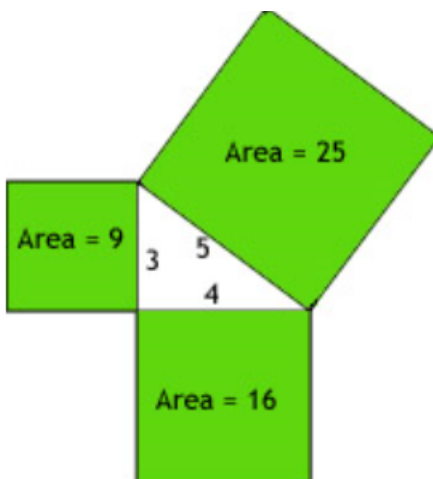
$$\sqrt{82}$$

Between 9 and 10

Pythagoras, a Greek mathematician, showed that in any right triangle, there is a special relationship among the **side lengths of a right triangle**. A right triangle is one that has two sides that form a **right angle (90°)**. The side opposite the right angle is called the **hypotenuse** and the two shorter sides are called **legs**.



He found that: *The area defined by the square of the hypotenuse equals the sum of the areas defined by the squares of the other two sides.*



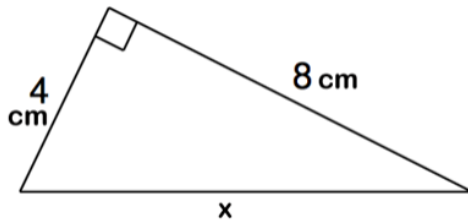
Hence,

$$5^2 = 3^2 + 4^2$$

$$25 = 9 + 16$$

Examples:

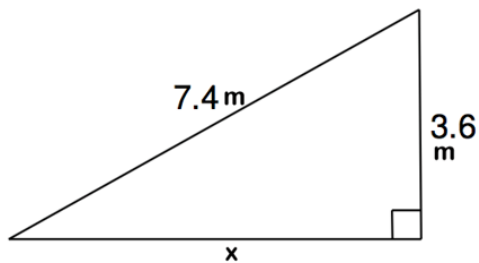
1. Solve for the unknown in each of these right triangles:



$$x^2 = 8^2 + 4^2$$

$$x = \sqrt{64 + 16}$$

$$x = \sqrt{80}$$



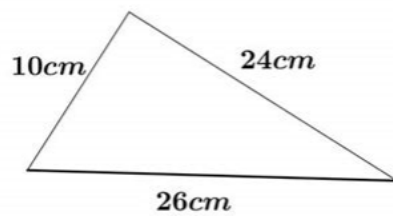
$$7.4^2 = x^2 + 3.6^2$$

$$54.76 = x^2 + 12.96$$

$$x^2 = 54.76 - 12.96$$

$$x = \sqrt{41.8}$$

2. Determine whether or not the following triangle has a right angle (Is it a **Pythagorean triplet?**):



$$26^2 = 10^2 + 24^2$$

$$676 = 100 + 576 \rightarrow \text{TRUE}$$

Any **whole** number multiplied by itself “twice” (three copies in total) will result in a **cube number**.

Perfect Cubes:

Number	Number Cubed	Perfect Cube
1	1^3	1
2	2^3	8
3	3^3	27
4	4^3	64
5	5^3	125
6	6^3	216
7	7^3	343
8	8^3	512
9	9^3	729
10	10^3	1000

**** You can use similar techniques for evaluating and estimating as you did with perfect squares and square roots ****

Examples:

1. Estimate:

$$\sqrt[3]{46}$$

Between 3 and 4

$$\sqrt[3]{24}$$

Between 2 and 3

2. Evaluate using calculator. Round answer to the nearest hundredth.

$$\sqrt[3]{50}$$

≈ 3.68

$$\sqrt[3]{74}$$

≈ 4.20

You can use the exponent laws to simplify an expression involving powers:

- To multiply powers with the same base, **add** the exponents
- To divide powers with the same base, **subtract** the exponents
- To raise a power to an exponent, **multiply** the exponents

Hence,

Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Raising a power to a Product	$(a \times b)^n = a^n \times b^n$
Raising a power to a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Zero Power	$a^0 = 1$
Negative Power	$a^{-1} = \frac{1}{a}$

Examples: Simplify

$$(a) (3^2)(3^4) = 3^{2+4} = 3^6 = 729$$

$$(b) 6^5 \div 6^3 = 6^{5-3} = 6^2 = 36$$

$$(c) (4^2)^5 = 4^{2 \times 5} = 4^{10} = 1,048,576$$

$$(d) (x^6)(x^5) = x^{6 \times 5} = x^{30}$$

$$(e) x^7 \div x^2 = x^{7-2} = x^5$$

$$(f) (x^5)^4 = x^{5 \times 4} = x^{20}$$

$$(g) (-2)^7(-2)^3 \div [(-2)^2]^3 = (-2)^{7+3} \div (-2)^{2 \times 3} = (-2)^{10} \div (-2)^6 = (-2)^{10-6} = (-2)^4$$

$$(h) \frac{(y^3)^5}{(y)(y^4)} = \frac{y^{15}}{y^5} = y^{15-5} = y^{10}$$

$$(i) -(453)^0 = -(1) = -1$$

$$(j) (2^3 \times 3^2)^2 = 2^{3 \times 2} \times 3^{2 \times 2} = 2^6 \times 3^4$$